Abstract argumentation for agent-based social simulations*

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Abstract. This paper discusses a possible application of computational argumentation for the social sciences. We show that an agent-based model of social behavior, where reasoning is grounded on Dung's abstract argumentation, can reproduce the micro-macro link of agreement without coordination in a population. Our agents are embedded in a social context, namely relational networks with different topologies. In each simulation, we monitor for the level of polarization among the interacting agents. We show that the level of polarization is influenced by information exchanged during argumentative dialogs between agents (our key micro-level assumption), and that network topologies aid the reaching of an agreement.

1 Introduction

Agent models have become an increasingly popular approach to social simulation. We can distinguish between two main streams of research: (*a*) a first stream which uses mathematical, game theoretical or evolutionary computing techniques; (*b*) a second stream which focuses on formal logic approaches.

The first stream of research focuses on agents that do, in fact, interact but where very little explicit reasoning is done. In the literature, agent reasoning is modeled in several possible ways:

- by linking the probability of an agent to choose between a set of opportunities by means of a threshold or continuous function or using function maximization to accomplish the same goal without invoking stochastic decision processes [20];
- by using theoretical games like the Iterated Prisoner's Dilemma to explore the dynamics of collectively beneficial actions that are costly to individuals in the short run but are a recurring feature of real social systems, and in particular to assess the emergence of a stable regime of cooperation between the (bounded) rational agents involved;

* This paper is an extended version of *Gabbriellini, S., Torroni, P.: Arguments in social networks (Extended Abstract). In: Proceedings of the 12th AAMAS Conference. IFAAMAS (2013)* [13]

– by using Genetic Algorithms to implement evolving strategies for signaling and detecting individual intentions in populations where agents face the decision to trust other partners, or to implement evolving collaborative or competitive strategies in Iterated Prisoner's Dilemma settings; or, finally, by using Neural Networks to explore social meta-reasoning and beliefs [10,2].

The second stream focuses explicitly on how agents should reason *socially*, i.e., interdependently with others, by means of formal logics [4,6]. The relevance of logic in ABSS is an open issue, with both detractors [9] and sustainers [5].

The striking fact is that BDI frameworks [23], like the ones advocated by Hedstrom [18], have not encountered a wide diffusion among sociologists, probably because most agent architectures based on the BDI paradigm are complex to understand and to use by non-computer-scientists, and often not suited for simulation with thousands of agents. The latter is the reason why the formal logic approach to ABSS has not simulated agents that interact socially to a significant extent in simulations (but see also Taillandiera [26]). On the other side, agents are mainly called *social* just because they are linked in network structures, but no reasoning is actually implemented.

In spite of a substantive claim for the adoption of agent-based models in the Analytical, Generative and Computational Sociologies [19,21,11], it looks like a shared framework to model key reasoning capabilities of social agents has not been developed yet. This paper aims at evaluating a new paradigm to model social agents which may result appealing for both streams of research in social simulation, thus filling a gap in the literature.

To model how agreement can be reached in discussions, we build on well established theories from social, cognitive, and computer sciences: the strength of weak ties by Granovetter [17], the argumentative nature of reasoning by Mercier & Sperber [22] and computational abstract argumentation by Dung [7]. The result is an agent-based model which simulates a population of social agents that interact within a relational structure, exchange information by means of simulated discussions and possibly reach an agreement. We show that the level of polarization is influenced by information exchanged during dialogs between agents (our key micro-level assumption), and that network topologies aid the reaching of an agreement.

There is already a plea for the use of logic-related approaches in ABSS [24], but we are not aware of any previous agent-based social simulations that uses argumentation to investigate agreement issues by simulating discussions with argumentative agents.

2 Embeddedness and social agents

Within social simulations, embeddedness in social simulations is almost always represented with (more or less explicit) network structures. Embeddedness could be something abstract, i.e. represented with relational networks, or spatial, i.e. represented with Von Neuman or Moore neighborhoods. In any case, these differ-

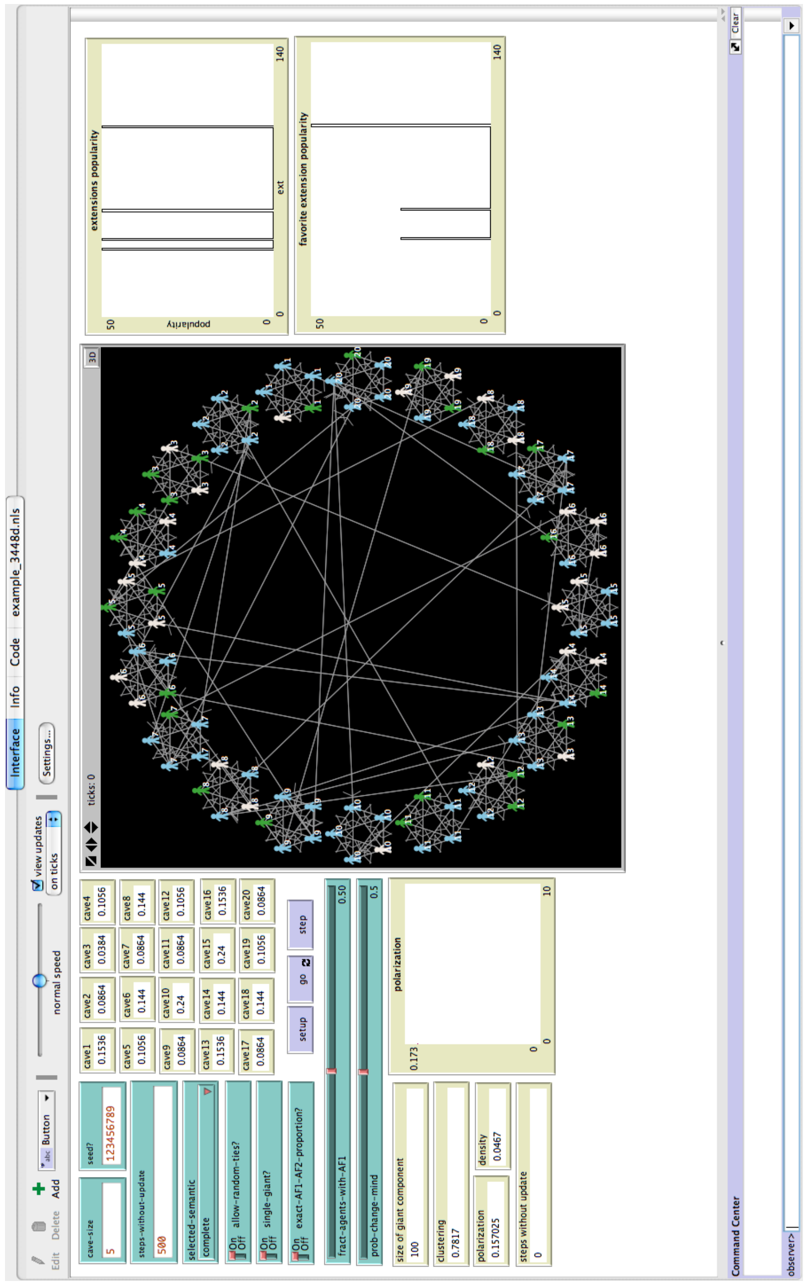

Fig. 1. The model NetArg at time 0, with bridges allowed in the caveman graph. The GUI shows on the right two plots where the extensions available at the population level are plotted.

ent kinds of embeddedness may all be explicitly represented by network topologies.

The basic idea of this social trait comes from Granovetter's hypothesis, which states that our acquaintances are less likely to be connected with each other than our close friends [17]. This tendency leads to social networks organized as densely knit clumps of small structures linked to other similar structures by bridges between them. The presence of a *bridge* implies that a person in one cluster is acquainted to a person in another cluster. Granovetter called this type of relation "weak tie", and demonstrated its importance in permitting the flow of resources, particularly information, between otherwise unconnected clusters [16].

Embeddedness and bridges express a network topology which exhibits high local clustering, i.e. clusters with high density, but at the same time low geodesic distances, i.e. the average shortest paths between any pair of nodes are short. Both these characteristics are observed in many real world settings and have been formalized by Watts & Strogatz [28] in the so-called "small-world" model.

Building on Granovetter's "strength of weak ties" theory [17], sociological research on "small world" networks suggests that in a social network the presence of bridges promotes cultural diffusion, homogeneity and integration, but only under the assumption that relations hold a positive value [12]. This last concern is a trademark of the social simulation stream which uses a non-reasoning approach to agent modeling. We will show that our model does not need such a specification.

Since our model is grounded in Granovetter's theory, we expect that the presence of bridges will lower the level of polarization among our interacting agents.

Following the experimental design by Flache & Macy [12], we use the "disconnected caveman graph" [27] to represent a situation where clusters are maximally dense. We then allow for two kind of structural settings:

- a first one where clusters are disconnected (thus becoming components of the graph) and agents are allowed to discuss only within their own "cave";[1]
- a second one where a random number of bridges is added between the clusters, in order to lower the geodesic distances in the whole network. Even if our mechanism does not guarantee that all the caves become connected, on average the resulting networks exhibit small-world network characteristics.

Such a network structure is imposed exogenously to agents and kept static once generated. Bridges are treated as weak links. By connecting previously unconnected densely knit caves, they play the role that acquaintances play in real life, and thus bridges are supposed to carry all the information beyond that available in a single cave.

However, we do not impose a positive or negative value to links. Instead, links only represent the possibility of communication between any two pair of agents. The bit of information transmitted may have a positive or negative value,

[1] A cave is a fully connected graph.

depending if the content of the exchange is something that reinforces agent's beliefs or that radically changes them.

We call the stream of information exchanged between two agent a "simulated dialogue". The dialogue mechanism represents the micro-level assumption that governs our model and builds on Mercier & Sperber's work.

3 Agents reasoning and interaction

According to Mercier & Sperber's argumentative theory of reasoning [22], the function of reasoning is argumentative and its emergence is best understood within the framework of the evolution of human communication. Reasoning developed as a "tool" to convince others by means of arguments exchanged while interacting with others. This is also the main reason why we are more proficient in solving a logical problem when it is in a real-world rather than in an abstract context.

Arguments may be more or less well formed and to avoid being victims of misinformation, addressees must exercise some degree of *epistemic vigilance*. Among others, two mechanisms seem to be particularly relevant for this scope: coherence setting and trust calibration. Mercier & Sperber describe these processes in detail, but for sake of brevity we report here only a brief summary:

1. Every time an addressee receives a new bit of information, she checks if it fits what she already knows. If this is the case, nothing happens, otherwise if the new information uncovers some incoherence, she has to react to avoid cognitive dissonance;
2. As an epistemically vigilant addressee, she faces two alternatives:
   - either to reject communicated information because the source is not trusted (or not trusted enough) to inject the belief revision mechanism. In this case, the addressee can produce an argument to attack the new information, defending her beliefs;
   - or to accept the new information because the source is trusted enough to start a coherence checking and allow for a fine-grained process of belief revision.
3. The source can react as well to the addressee's reaction at level two: if the addressee decides to refuse the new information, the source can produce arguments to inject trust in the addressee, like exhibiting a social status which demonstrates competences on the subject matter. Otherwise, the source can produce arguments to persuade the addressee that the new information is logical and coherent, or to rebut the addressee's attack.
4. Both addressee and source take the risk of revising their own beliefs while involved in such a turn-taking interaction where point two and three are repeated until:
   - addressee or source revises her own beliefs;
   - addressee or source does not trust the other, thus nothing changes.

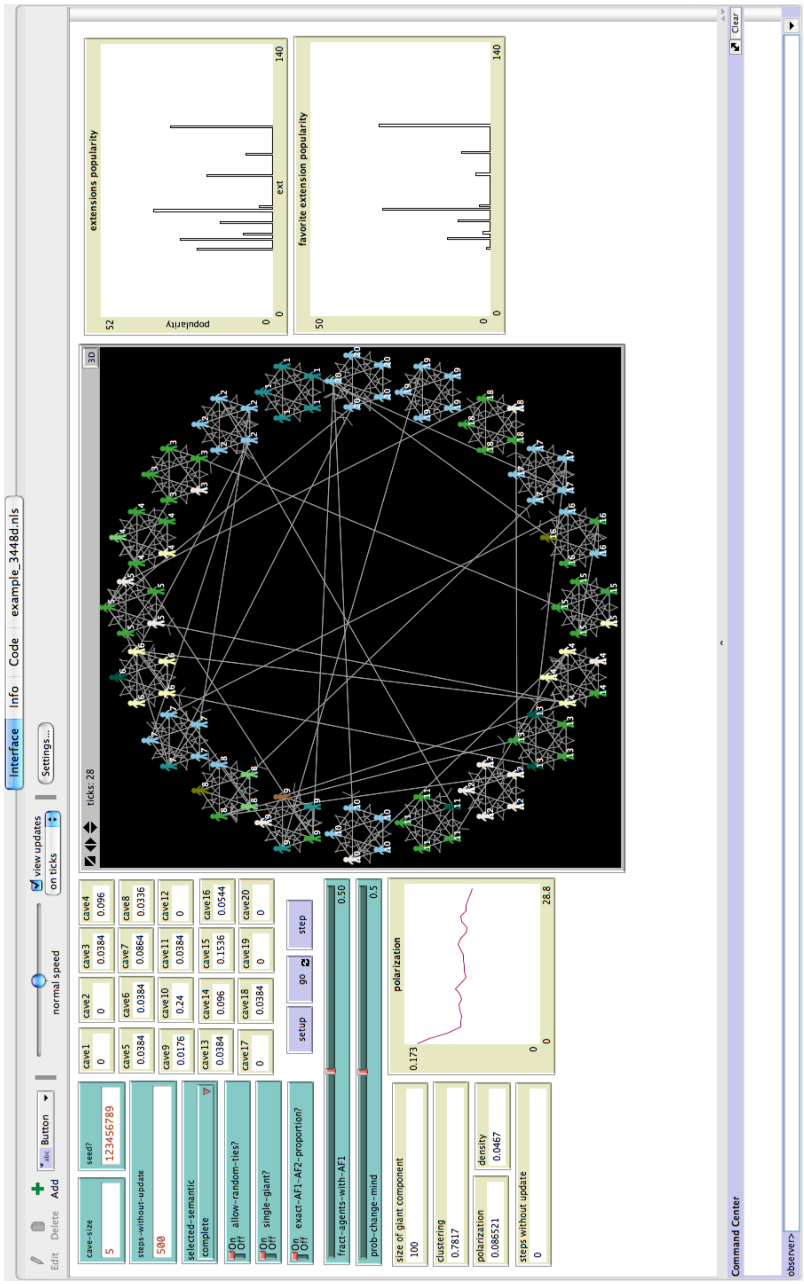

Fig. 2. New beliefs emerge from interaction in dialogues.

Such a turn-taking interaction between communicants is called a "dialogue". In a dialogue, if a highly trusted individual tells us something that is incoherent with our previous beliefs, some revision is unavoidable. On the other hand, if a communicator wants to communicate a piece of information that the addressee is unlikely to accept on trust, she can produce arguments for her claims, and encourage the addressee to examine, evaluate, and accept these arguments.

What we believe is a result of a reasoning process which takes as input the arguments we know and their relations. We could represent the essence of our beliefs as a network of relations between the arguments we know. If we restrict such a network to attack relations only, such a conceptual framework allow us to introduce computational argumentation, and in particular abstract argumentation, as the key reasoning capability of our artificial agents.

In computational abstract argumentation, as defined by Dung [7], an "Argumentation Framework" (AF) is defined as a pair $\langle \mathcal{A}, \mathcal{R} \rangle$, where $\mathcal{A}$ is a set of atomic arguments and $\mathcal{R}$ is a binary *attacks* relation over arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$, with $\alpha \rightarrow \beta \in \mathcal{R}$ interpreted as "argument α attacks argument β." Sets of "justified" arguments can be described by various extension-based semantics [3]. In particular, an extension-based semantics identifies a number of subsets of $\mathcal{A}$ that all together represent a coherent set of beliefs.

Argumentation semantics may be conflict-tolerant, whereby arguments in the same extension may attack each other [1], but most proposals require that extensions be *conflict-free*, i.e., no two arguments in the same extension may attack each other. In our model, we use conflict-free semantics. Therefore if an extension contains an argument α, either $\mathcal{R}$ does not contain any attack to α, or else α is defended, from all incoming attacks, by arguments in the extension. Not all the semantics guarantee that there is always an extension. In fact, in general a semantics may define for a given AF one extension, more than one, or even none at all. For example, according to many semantics, such as the *complete* semantics [7] and the *ideal* semantics [8], an argumentation framework such as $\langle \{a, b\}, \{a \rightarrow b, b \rightarrow a\} \rangle$ admits at least two extensions: $\{a\}$ and $\{b\}$. Other semantics, such as the *grounded* semantics [7], require that there is only one extension. According to the grounded semantics, the only extension for the above AF is the empty set.

We thus propose an agent-based model for simulating interaction between social agents by means of abstract arguments exchanged in simulated dialogues [13]. Agents reason argumentatively, and implement epistemic vigilance by way of trust calibration and coherence setting within a dialogue.

For the sake of generality, several choice points are left open. Mainly, we do not commit to any specific argumentation semantics, we do not commit to any specific belief revision mechanism, and we do not specify how trust is formed. About the latter, we assume that agents rely on a trust model. Arguably a realistic model of trust would need to be a dynamic measure that takes into account the *nature of social ties*, and the *authoritativeness*, *expertise* and *social status* of the interlocutor [25]. However, our dialogue model is orthogonal to the trust model, and different trust models can be accommodated.

Algorithm 1 Simulate an iteration of the model.

Require: $N_I > 0$ {N_I is the number of iterations}
Require: $N_A > 0$ {N_A is the number of agents}
  for $I = 1 \rightarrow N_I$ **do**
    for $A = 1 \rightarrow N_A$ **do**
      select a random agent B within A's neighbors
      initiate dialog with B
    end for
    record statistics
  end for

The process that maps human beliefs onto argumentation frameworks and human interaction onto simulated dialogues is explained in more detail in a companion paper [14]. Briefly, a simulated dialogue starts with an "invitation to discuss" from A (communicator) to B (addressee), by picking a random argument σ in its own extension. This sparks a dialog $\mathcal{D}$. B then evaluates σ by trust calibration/coherence setting. During $\mathcal{D}$, A and B establish the coherence of claims against their own beliefs by argumentative reasoning. If σ is incoherent with B's beliefs, and B trusts A, B will actuate some form of belief revision in order to be able to include σ among her beliefs, while maintaining coherence. Otherwise, if σ is incoherent with B's beliefs, and B does not trust A, B will engage in a dialog with A, by producing arguments against σ. Similarly, A can produce arguments for her claims, and encourage B to examine, evaluate, and accept these arguments.

Whenever B is addressed by A with $\mathcal{D}$, B evaluates σ. If σ is coherent with B's AF, the dialog has no reason to continue: A and B agree on the subject and the dialog ends. If σ is incoherent with B's AF, i.e., if σ is not included in any of B's extensions, B will use mechanisms to exercise epistemic vigilance. In particular, if B *trusts* A, B will believe what A says, and revise her own beliefs (i.e., her argumentation framework) in order to accommodate A's argument σ in at least one of her extensions. If instead B does not trust A, B will produce an argument α against σ and wait for a reaction from A.

Agents can revise their beliefs by learning an attack between two arguments and thus update their extensions. For the sake of simplicity, we define trust thresholds statically. The exchange between A and B continues until one of the agents changes her mind (agreement is thus reached), or if both agents leave the dialogue because neither is persuaded.

4 Simulation model and experiments with NetArg

We developed our model NetArg [15] using NetLogo [29] and wrote a NetLogo module to deal with the computational argumentation analysis.

The model, shown in Figure 1, comprises a fixed number of agents (100) distributed in 20 distinct caves. Each agent has an argumentation framework (AF) where a number of arguments and a certain number of attacks between arguments are present. We set up each experiment by distributing, with different probability among the population, two alternative AFs. At each time step, each agent is asked to start a dialogue with one of her neighbors extracted at random (see Algorithm 1), who could be restricted to the same cave or not, depending on the presence of bridges. The random extraction assures that the probability to "argue" with members of the same cave is higher than with out-cave neighbors, according to the fact that weak ties (bridges) are less activated than strong ties.

The agent selected to start a dialogue picks one random argument in her extensions (i.e., an argument she believes in) and addresses the previously selected neighbor. The opponent replies following the dialogue procedure briefly sketched above. The dialogue ends if either agent changes her mind (agreement is thus reached), or if both agents leave the dialogue because neither is persuaded to revise beliefs.

It is evident from the two plots on the right in Figure 2 that, after some steps, agents adopt new beliefs by means of dialogues (now more bars are present compared to Figure 1, where each bar represents the popularity of a particular extension). This happens because agents exchange attacks between the arguments they know, and these attacks, if accepted, may call for a belief revision in the AF of the addressees.

The belief revision process gives raise also to a polarization effect at the population level. By polarization we mean that a population divides into a small number of factions with high internal consensus and strong disagreement between them. A perfectly polarized population contains two opposing factions whose members agree on everything with each other and fully disagree on everything with the out-group.

Using a modified version of the measure used by Flache & Macy [12], we measure the level of polarization P at time t as the variance of the distribution of the AF distances $d_{ij,t}$:

$$P_t = \frac{1}{N(N-1))} \sum_{i \neq j}^{i=N, j=N} (d_{ij,t} - \gamma_t)^2$$

where:

- N represents the number of agents in the population;
- $d_{ij,t}$ represents the AF distance between agents i and j, i.e., the fact that agent i has an argument in her semantic extension ($\bigcup_{\mathcal{E}}^{i}$) while the other does not, averaged across all available arguments ($|\mathcal{A}|$):

$$d_{ij} = \frac{|\bigcup_{\mathcal{E}}^{i} \setminus \bigcup_{\mathcal{E}}^{j} \cup \bigcup_{\mathcal{E}}^{j} \setminus \bigcup_{\mathcal{E}}^{i}|}{|\mathcal{A}|}$$

- γ_t represents the average distance value at time t;

We present here an experiment which aims at testing if the model can reproduce Granovetter's theory about weak ties: does the model exhibit a long-range ties effect on social polarization? Or, to rephrase the question, does the presence of weak ties (i.e. bridges) lower polarization at the population level?

In this experiment, we use two AFs made by 5 arguments ($\mathcal{A} = \{a, b, c, d, e\}$) with these attacks relations that abstract away two positions derived from a real debate in an online discussion forum about renewable energies[2]:

- $AF_1 = \langle \mathcal{A}, \{c \rightarrow b, d \rightarrow a, e \rightarrow d\} \rangle$
- $AF_2 = \langle \mathcal{A}, \{b \rightarrow a, b \rightarrow c, b \rightarrow e, d \rightarrow a\, d \rightarrow e\} \rangle$

We set the AFs distribution fixed at 0.5 and allowed trust (*prob-change-mind* parameter in the GUI) to take these values: 0, 0.2, 0.5, 0.8 and 1. For each parameter value, we ran the model 100 times. We tested the model also on random AFs up to 6 arguments and found that this result is stable.

The results displayed in Figure 3 are averages for each parameter combination. From what we observe in the plot, only the presence of bridges lowers polarization. During a run, dialogues enhance polarization because they give raise to new beliefs sets, thus increasing “cultural” distance among the agents.

With no bridges connecting caves (a), each cave quickly stabilizes at a local minimum. However, in general different caves will end up in different local minima, which results in a high polarization overall. Trust is able to lower the curve, but only until 0.8, because at 1 every agent changes her mind continuously so that polarization is even enhanced. In a sense, agents with total trust are “gullible” agents ready to believe anything. The instability arises if *all* agents are gullible, because there is no stable opinion.

On the contrary, when bridges are present (b), polarization levels are lowered considerably. This time, caves can receive information from other caves, and this “small-world” topology lets the population exit from local minima. Increasing trust is more effective in this case, and values as low as 0.5 are able to lower polarization nearly to 0.

We can conclude that the model fits the predictions of Granovetter’s theory: (1) the presence of bridges between caves fosters agreement and consensus, growing the number of “like-minded” agents and (2) since only caves with bridges to other caves can receive new information, only connected caves learn new relations between arguments and change their minds.

5 Conclusions

In this paper we demonstrated that computational argumentation can be profitably used in ABSS in order to obtain significant results with an artificial population of argumentative agents.

From a theoretical standpoint, we found that our hypotheses on the dialogue procedure are, in principle, sufficient to reproduce two macro-behaviors embedded in Granovetter’s theory, i.e., the tendency to inclusion of weak ties and a competitive advantage for non-isolated caves. From a methodological standpoint,

[2] `http://www.energeticambiente.it`

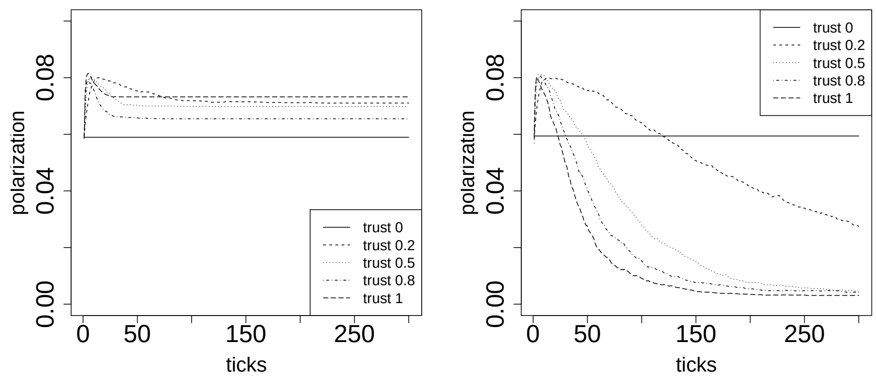

Fig. 3. Average polarization levels (over 100 runs) for two conditions: (a) without random ties, (b) with random ties. AFs distribution is 0.5. Different levels of trust are shown.

our model is–at least in principle–more expressive than other models from the literature where opinions are expressed in a linear scale. At the same time, it is not as complex as BDI models.

To the best of our knowledge, our proposal is original both in the social sciences, where computational argumentation has never been used for social simulation, and in agent research, where argumentation-based interaction has been thoroughly investigated, but without extra-logical elements, such as trust, which are important from a cognitive standpoint. Moreover, argumentative agents have been so far deployed only at a small scale. Our simulation framework can be used to better understand the behavior of argumentation semantics when large populations of agents are involved. For example, from some experiments not reported here, it seems that some AF structures are more resistant to external modifications than others. It would be interesting to discover patterns, strengths and weaknesses of such structures, both from an engineering perspective (e.g., for building robust artificial agents) and from a social science perspective (e.g., to understand which argumentation semantics better model human behavior, and if/why some opinions are stronger than others in a social debate).

Our main future goal is to understand if the model is able to forecast the outcome of a discussion by simulating a virtual discussion which starts from similar premises. This is the reason why we think that our work finds useful applications not only in theoretical research, but also in the domains of interest of policy-makers, like sustainable energy, political discussions and e-participation.

In particular, this work has been developed in the context of the ePolicy EU project,[3] whose main aim is to support policy makers in their decision process

[3] `http://www.epolicy-project.eu/`

across a multi-disciplinary effort aimed at engineering the policy making lifecycle. By simulating with argumentative agents, we could help policy-makers understand how a topic is being discussed, what positions (arguments) are involved in a debate, and how they relate with one another. Ultimately, by simulation, we could forecast a range of possible conclusions that may emerge from such debates. To that end, we need to massively extend the replication of empirical data in order to test the soundness of our approach to real-world settings.

Acknowledgments

This work was partially supported by the ePolicy EU project FP7-ICT-2011-7, grant agreement 288147. Possible inaccuracies of information are under the responsibility of the project team. The text reflects solely the views of its authors. The European Commission is not liable for any use that may be made of the information contained in this paper.

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